NUMERICAL ANALYSIS OF INTERMODULATION DISTORTION IN MICROWAVE MIXERS.

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ABSTRACT

The paper introduces a general-purpose program for intermodulation distortion analysis in microwave mixers. The program can perform full nonlinear simulations of arbitrarily defined circuits simultaneously excited by three independent sinusoidal sources. The analysis relies upon a three-dimensional sampling of the signal waveforms coupled with a triple Fourier transform.

INTRODUCTION

The analysis of microwave mixers by the conversion-matrix technique (CMT) is a well-established procedure {1-6}, and has recently been extended to the treatment of intermodulation distortion {7}. This method offers considerable generality and flexibility coupled to moderate computer time requirements, which makes for its easy implementation even on small-size systems. On the other hand, the CMT relies upon a linearization, and is thus subject to a number of restrictions {8}, the most important one being represented by the fact that saturation effects cannot be dealt with in this way.

For this reason, several investigators have recently approached the mixer problem by a full nonlinear analysis based on the harmonic-balance (HB) concept implemented in one {9-12} or two {13} dimensions. When it comes to intermodulation distortion analysis, such techniques are usually not adequate. In this case the circuit is excited by three sinusoidal signals, the local oscillator (LO) and two independent radio frequencies (RF). The difference between the latter is most often very small with respect to the RF and LO frequencies, so that the required number of sampling points may easily exceed the memory resources and the computational capabilities of even the largest available mainframes. In order to overcome such difficulties, in this paper we analyze mixer intermodulation by an HB technique using three-dimensional sets of sampling points and triple Fourier transforms to perform time-to-frequency-domain conversions. The approach has been implemented in a general-purpose CAD environment {14}, allowing arbitrary passive circuit topologies and device models to be dealt with.

The present method offers several distinctive advantages over its predecessors. It can work with any signal amplitude, which is essential in order to determine the mixer dynamic range, with respect to both conversion-gain compression and IMD products level. Its numerical performance (including memory occupation and CPU time) is independent of the actual frequency values used in the simulation of a given circuit. It is conceptually rigorous, which makes it an ideal reference for establishing the accuracy of any approximate procedure that might be devised to solve the same problem.

INTERMODULATION ANALYSIS VIA TRIPLE FOURIER TRANSFORM

Let us consider a nonlinear circuit excited by three independent sinusoidal signals of angular frequencies ω_1, ω_2, ω_3. In steady-state conditions the spectrum of any

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time-dependent quantity (such as voltages and currents) will contain all possible IMD products of the fundamentals, namely, all spectral lines of angular frequencies

$$\omega = n_1 \omega_1 + n_2 \omega_2 + n_3 \omega_3 \quad (1)$$

where n_1, n_2, n_3, are arbitrary integers. For mixer intermodulation analysis, it is more convenient to rewrite (1) in the form

$$\omega = n_O \omega_1 + n_2(\omega_2 - \omega_1) + n_3(\omega_3 - \omega_1) \quad (2)$$

where ω_1 is given the meaning of LO frequency and ω_2, ω_3 are independent radio frequencies. The integer coefficients appearing in (2) satisfy the inequalities

$$0 \le |n_O| \le N_O \qquad 0 \le |n_2| + |n_3| \le M \quad (3)$$

where M is the maximum order of intermodulation products of ω_2, ω_3 to be accounted for. In the commonly encountered case of $|\omega_2 - \omega_1| << \omega_1$ and $|\omega_3 - \omega_1| << \omega_1$ the spectrum consists of $M^2 + M + 1$ baseband (IF) lines plus N_O clusters of $2M^2 + 2M + 1$ lines each, centered around the LO harmonics.

In order to analyze the circuit by the piecewise harmonic-balance technique {15}, one has to generate the frequency-domain response of the nonlinear subnetwork to an excitation of the general form

$$x(t) = \sum_{n_1,n_2,n_3} X_{n_1 n_2 n_3} \exp[j(n_1\omega_1 + n_2\omega_2 + n_3\omega_3)t] \quad (4)$$

To do so, the quantities $z_i = \omega_i t$ (i = 1, 2, 3) appearing in (4) are considered as *independent* variables, so that the time-domain response becomes a 2π-periodic function of each of the z_i. The spectral components may thus be found by a three-dimensional Fourier transform.

As a first step, a three-dimensional grid of sampling points is created by computing the nonlinear subnetwork response for the following values of the independent time variables (in any combination):

$$z_i = \omega_i t = (r_i - 1) \frac{2\pi}{N_i} \qquad 1 \le r_i \le N_i, \quad i = 1, 2, 3 \quad (5)$$

where N_i is the number of sampling instants in the z_i dimension. When the spectrum is defined by (2), (3), according to the sampling theorem one must have

$$N_1 > 2(N_O + M) \qquad N_i > 2M, \quad i = 2, 3 \quad (6)$$

Note that, unlike a conventional IMD analysis, the mixer case requires a rectangular set of sampling points.

Once the samples of the time-domain response have been found, its harmonics can be determined by a triple FFT. Quite obviously this calculation could be performed by an iterated application of the one-dimensional transform, but it is much more convenient to make use of sophisticated computational schemes that directly adress the multidimensional case. For instance, the algorithm described by Nobile and Roberto {16} allows FFT costs to be cut by a factor of about 6 when run on a Cray X-MP computer in the typical case $N_1 = 16$, $N_2 = N_3 = 8$. In turn, this results in an average 40% reduction of the overall circuit analysis cost.

The calculation then proceeds as in a conventional harmonic-balance analysis {15}.

A NUMERICAL EXAMPLE

As an example of the program capabilities, we report in this section the results of a mixer intermodulation analysis of practical interest. We consider a single-ended FET gate mixer, whose topology is reported elsewhere {13}, pumped by a 6 dBm LO at $f_1 = 8$ GHz

($f = \omega/2\pi$), and assume that two RF signals of equal amplitudes (-15 dBm), and frequencies f_2= 8.5 GHz, f_3= 8.51 GHz are fed to the mixer input. The intermodulation analysis is carried out with 4 LO harmonics and takes into account all IMD products of f_2, f_3 up to the 3rd order, for a total of 113 frequencies. The resulting drain current spectrum is given in Table I below. All figures listed in the table are numerically exact. Note that the spectral lines are identified by 3 harmonic numbers k_1, k_2, k_3 ($f = k_1 f_1 + k_2 f_2 + k_3 f_3$). The calculation takes about 19 CPU seconds on a Cray X-MP/48.

The results shown in Table I give a clear account of the impressive amount of data that a typical mixer IMD analysis has to deal with. Note that the reasonable CPU time implies that the same analysis could easily be run on a medium-size scalar computer such as a VAX system.

ACKNOWLEDGMENTS

This work was supported by the Italian National Research Council (CNR) and by the Istituto Superiore delle Poste e delle Telecomunicazioni (ISPT). The development of vectorized software tools for nonlinear microwave CAD is currently being pursued as a joint research effort by the Electronics Department of the University of Bologna, Fondazione Ugo Bordoni, and Fondazione Guglielmo Marconi. The authors gratefully acknowledge the contribution of Drs. Nobile and Roberto of SISSA (Trieste - Italy) who provided the vectorized multidimensional FFT routines used herein.

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TABLE I - Drain current spectrum in a FET gate mixer

Harmonic numbers			Frequency	Amplitude
k_1	k_2	k_3	(GHz)	(mA)
0	0	0	0.00	0.34539E+2
0	-1	1	0.01	0.26728E-1
-1	2	-1	0.49	0.13735E-1
-1	1	0	0.50	0.30552E+1
-1	0	1	0.51	0.30558E+1
-1	-1	2	0.52	0.13720E-1
-2	2	0	1.00	0.54613E-1
-2	1	1	1.01	0.10994E00
-2	0	2	1.02	0.54502E-1
-3	3	0	1.50	0.12223E-2
-3	2	1	1.51	0.36282E-2
-3	1	2	1.52	0.36350E-2
-3	0	3	1.53	0.12324E-2
4	0	-3	6.47	0.50574E-3
4	-1	-2	6.48	0.14898E-2
4	-2	-1	6.49	0.14917E-2
4	-3	0	6.50	0.50721E-3
3	0	-2	6.98	0.71738E-2
3	-1	-1	6.99	0.13506E-1
3	-2	0	7.00	0.71825E-2
2	1	-2	7.48	0.38264E-2
2	0	-1	7.49	0.40377E00
2	-1	0	7.50	0.40387E00
2	-2	1	7.51	0.38325E-2
1	1	-1	7.99	0.94440E-1
1	0	0	8.00	0.51885E+2
1	-1	1	8.01	0.80320E-1
0	2	-1	8.49	0.10810E-1
0	1	0	8.50	0.46329E+1
0	0	1	8.51	0.46310E+1
0	-1	2	8.52	0.97681E-2
-1	2	0	9.00	0.12440E-1
-1	1	1	9.01	0.25384E-1
-1	0	2	9.02	0.12667E-1
-2	3	0	9.50	0.45767E-3
-2	2	1	9.51	0.12460E-2
-2	1	2	9.52	0.12499E-2
-2	0	3	9.53	0.45862E-3
5	0	-3	14.47	0.11681E-2
5	-1	-2	14.48	0.32154E-2
5	-2	-1	14.49	0.32185E-2
5	-3	0	14.50	0.11745E-2
4	0	-2	14.98	0.22068E-1
4	-1	-1	14.99	0.44236E-1
4	-2	0	15.00	0.22143E-1
3	1	-2	15.48	0.52968E-2
3	0	-1	15.49	0.58720E00
3	-1	0	15.50	0.58915E00
3	-2	1	15.51	0.55071E-2
2	1	-1	15.99	0.10560E00
2	0	0	16.00	0.19026E+2
2	-1	1	16.01	0.10267E00
1	2	-1	16.49	0.20098E-2
1	1	0	16.50	0.26996E+1
1	0	1	16.51	0.26982E+1
1	-1	2	16.52	0.71163E-4
0	2	0	17.00	0.70260E-1
0	1	1	17.01	0.14082E00
0	0	2	17.02	0.70178E-1
-1	3	0	17.50	0.91428E-3
-1	2	1	17.51	0.30758E-2
-1	1	2	17.52	0.31009E-2
-1	0	3	17.53	0.94515E-3
6	0	-3	22.47	0.53277E-3
6	-1	-2	22.48	0.15940E-2
6	-2	-1	22.49	0.15954E-2
6	-3	0	22.50	0.53321E-3
5	0	-2	22.98	0.99681E-2
5	-1	-1	22.99	0.21008E-1
5	-2	0	23.00	0.99665E-2
4	1	-2	23.48	0.20923E-2
4	0	-1	23.49	0.14149E-1
4	-1	0	23.50	0.15241E-1
4	-2	1	23.51	0.18513E-2
3	1	-1	23.99	0.30830E-1
3	0	0	24.00	0.50699E00
3	-1	1	24.01	0.30343E-1
2	2	-1	24.49	0.10170E-1
2	1	0	24.50	0.34582E-1
2	0	1	24.51	0.33345E-1
2	-1	2	24.52	0.89706E-2
1	2	0	25.00	0.95120E-2
1	1	1	25.01	0.20432E-1
1	0	2	25.02	0.99640E-2
0	3	0	25.50	0.16350E-2
0	2	1	25.51	0.49648E-2
0	1	2	25.52	0.49768E-2
0	0	3	25.53	0.16607E-2
7	0	-3	30.47	0.67712E-3
7	-1	-2	30.48	0.18153E-2
7	-2	-1	30.49	0.18209E-2
7	-3	0	30.50	0.68594E-3
6	0	-2	30.98	0.63915E-2
6	-1	-1	30.99	0.11949E-1
6	-2	0	31.00	0.63991E-2
5	1	-2	31.48	0.26135E-2
5	0	-1	31.49	0.12592E00
5	-1	0	31.50	0.12647E00
5	-2	1	31.51	0.26534E-2
4	1	-1	31.99	0.51017E-1
4	0	0	32.00	0.19985E+1
4	-1	1	32.01	0.50530E-1
3	2	-1	32.49	0.70936E-2
3	1	0	32.50	0.55818E00
3	0	1	32.51	0.55796E00
3	-1	2	32.52	0.66859E-2
2	2	0	33.00	0.52027E-1
2	1	1	33.01	0.10503E00
2	0	2	33.02	0.52117E-1
1	3	0	33.50	0.19182E-2
1	2	1	33.51	0.56297E-2
1	1	2	33.52	0.56348E-2
1	0	3	33.53	0.19417E-2